Forecasting Time-Varying Correlation using the DCC Model

John D. Eustaquio, Dennis S. Mapa, Miguel C. Mindanao, and Niño I. Paz
School of Statistics
University of the Philippines Diliman

Hedging strategies have become more and more complicated as assets being traded have become more interrelated to each other. Thus, the estimation of risks for optimal hedging does not involve only the quantification of individual volatilities but also include their pairwise correlations. Therefore a model to capture the dynamic relationships is necessary to estimate and forecast correlations of returns through time. Engle's dynamic conditional correlation (DCC) model is compared with other models of correlation. Performance of the correlation models are evaluated in this paper using only the daily log returns of the closing prices from January, 2000 to February, 2010 of the Peso-Dollar Exchange Rate and Philippine Stock Exchange index. Ultimately, Engle's DCC model is adopted because of its consistency with expectations. Though generally negative, correlation between these two returns is not really constant as the results indicated. The forecast evaluation of the models was divided into in-sample and out-of-sample forecast performance with short-term (i.e., 22-day, 60-day, and 125-day) and medium-term (250-day and 500-day) rolling window correlations, or realized correlations, as proxies for the actual correlation. Based on the root mean squared error and mean absolute error, the integrated DCC model showed optimal forecast performance for the in-sample correlation patterns while the mean-reverting DCC model had the most desirable forecast properties for dynamic long-run forecasts. Also, the Diebold-Mariano tests showed that the integrated DCC has greater predictive accuracy in terms of the 3-month realized correlations than the rest of the models.

Keywords: dynamic conditional correlation, peso-dollar exchange rate, PSE index, hedging

1. Introduction

Correlations are vital inputs for many of the tasks of financial risk management. Forecasts of future correlations and volatilities are the basis of any pricing formula for financial instruments or strategy that would aid an investor or a company in mitigating unnecessary risks. Thus, it has been a tale of the tape to efficiently and accurately forecast financial correlations for risk planning and policy-making purposes. Several models have already been developed to capture the correlation pattern of financial time series. Some of the most common models are the constant conditional correlation model, the diagonal vector generalized autoregressive conditional heteroskedastic (VECH) model, and the diagonal Baba, Engle, Kraft and Kroner (BEKK) model, which are multivariate generalized autoregressive conditional heteroskedastic (GARCH) methods readily estimated in latest versions of most software packages. However, Engle (2001) proposed a model that addresses the structural and empirical weaknesses of the latter models and attempts to accurately and parsimoniously pin down the dynamic pattern of the correlation which is why it is aptly name as the dynamic conditional correlation (DCC) model. The goal is to assess the performance of the DCC model in forecasting the correlations between the returns of two Philippine financial series – the Philippine Stock Exchange Index (PSEi) and the nominal Peso-Dollar exchange rate– and see how the results compare with other correlation models.

Hedging problem

One way that investors and institutions can manage the risks they're entering is through hedging, that is entering into another transaction whose sensitivity to fluctuations in prices counterbalances the sensitivity of their main transaction to such variations. A common hedging strategy is optimization of portfolio allocation. This entails minimizing the variance of the portfolio as it measures the level of risk of a pool of investments. The variance of a given portfolio is a function of the individual variability of each investment and their correlations with each other. Another hedging strategy commonly employed is the use of derivatives. Derivatives are basically contracts that are valued based on the price of other assets. They protect the holder of the contract by giving a payoff once a stipulated event occurs, usually a significant drop of the price level relevant to the main transaction, at a specific point of time. This protection however, comes with a premium to be paid at the commencement of the derivative, thus this financial instrument are almost synonymous with insurance.

An important hedging problem that arises involving the PSE Index and the Peso-Dollar exchange rate is usually encountered by a foreign investor wishing to buy shares in the Philippines using a US Dollar currency. In entering such a transaction, the investor not only exposes himself to the risk of a sudden decline in the level of the PSEi but also exposes himself to the risk of the Dollar depreciating against the Peso. An equity-linked forward, called a quantity adjusting option (quanto), is a derivative instrument that is designed to give a Dollar payoff if the overall Dollar value of the investment (the quotient of the PSEi and the Peso-Dollar exchange rate times the number of shares purchased) at the end of the quanto's expiration is less than some predetermined level. Although the original investment did not merit a profit, the investor was able to insure himself for the possible downfall. However, a seller determining a fair price for the quanto involves information on several financial parameters such as volatilities, interest rates, exchange rates, and most importantly, the correlation between the PSE index and the Peso-Dollar exchange rate.

2. Forecasting Correlations

Movements in the levels of financial series such as the PSEi and the Peso-Dollar exchange rate are very much determined by internal and external news that are relevant to them. This news includes government policy implementations, interventions in markets, inside information, and recessionary movements. Prices of assets constantly change in response to news and in anticipation of future performance. Moreover, if news affecting both assets are correlated, then the prices of these assets will also be correlated. Then, both the volatilities of and correlations between asset returns or prices will depend on the information to update their distribution (Samuelson, 1965). Because of the existence of this correlation, one can generally dictate how the price or return of an asset will move based on the movement of the other asset. However, the nature of the news that affect both assets also vary across time. Thus, it is proper to think that the correlations between these assets are also varying across time. It should also be noted that forward-looking correlations are considered more important for an investor as this will dictate how the investment should be structured so that expected future payoffs will be realized. Thus, it might be of advantageous to investors if accurate long-term correlations could be forecasted so that proper strategies can be employed.

Though it is hypothesized that the correlations between financial series are time varying, several economic theories are proposed to give a general idea of the nature of these correlations. Macroeconomic theory suggests that in a healthy economic outlook, it is expected to have high equity levels and an appreciating currency, controlling for other variables. Cappiello and De Santis (2005) postulates the uncovered equity return parity condition which states that a country with a lower expected equity return will have an appreciating currency with respect to the country with a higher expected equity return.

In order to quantitatively capture the time-varying correlations between asset returns, several statistical models have been proposed various literatures.

The Constant Conditional Correlation (CCC) model (Bollerslev, 1990) is a class of multivariate GARCH models which restricts the correlation matrix to be time invariant such that $H_t = D_t R_t D_t$ where H_t is the conditional covariance of two asset returns at time t and R_t is the conditional correlation matrix of two asset returns at time t. Though the estimation is simple, one major drawback is that nothing much can be learned about the dynamics of the correlations while assuming they are constant, though some studies have pointed out a constant correlation pattern in some short-term periods. A more popular class of multivariate GARCH models is the diagonal multivariate GARCH or diagonal VECH which formulates the i,j element of the covariance matrix as the product of the prior i and j returns (Bollerslev, Engle, & Wooldridge, 1988). The first-order diagonal VECH model is given by

$$h_{i,j,t} = \omega_{i,j} + \alpha_i \alpha_j r_{i,t-1}\, r_{j,t-1} + \beta_i \beta_j h_{i,j,t-1}$$

However, the diagonal VECH model does not guarantee the positive definiteness of the resulting covariance matrices. The diagonal vector GARCH model was eventually modified to guarantee positive semi-definiteness of the covariance matrices by imposing a simple restriction for all the α's to be the same and all the β's to be the same, and requires that the matrix containing the elements $\omega_{i,j}$ to be positive definite. The model is called the diagonal BEKK vector GARCH model (Engle & Kroner, 1995) and it is given by:

$$h_{i,j,t} = \omega_{i,j} + \alpha r_{i,t-1}\, r_{j,t-1} + \beta h_{,j,t-1}$$

It should be noted that for the diagonal VECH and diagonal BEKK models, the model specifies the dynamic pattern of the covariances, not the correlations, though they can be computed as an ad hoc procedure.

A model that directly estimates the conditional correlation is the dynamic conditional correlation model (Engle, 2002). This model formulates the volatilities of returns in one set of equations and the correlations between them in another set thus, treating them as independent stochastic processes, entailing more flexibility and different parameterizations. From Monte Carlo simulations and real data context applications of the dynamic conditional correlation (DCC) model, it yielded the smallest mean absolute error among the vector GARCH and vector BEKK models, pinning much promise as to the quality of the results and simplicity of the method.

Many authors have used the DCC model to investigate the correlations of several macroeconomic variables. Bautista (2003) used the model to analyze the dynamic relationship between interest rate and exchange rate in the Philippines and saw that the correlations are mainly due to the effects of policies to exogenous events. Also, Vargas (2010) used a DCC model with an exogenous predictor to determine the key drivers of correlations between equity returns and exchange rate in select countries in Europe. Results show that interest rate differentials and capital flows were significant in explaining correlations between equity returns and exchange rate.

3. The Dynamic Conditional Correlation Model

The DCC was used to model the time-varying correlations between the returns of the PSEi and the exchange rate. This model is commonly used to understand changes in correlations in asset returns by treating them as random processes. In this model, the correlation estimates are updated every time new information on volatility-adjusted returns arrives (Engle, 2002).

The first-difference of the logarithm of the prices of each asset was taken to extract the log returns of the PSEi and exchange rate. The DCC model requires the standardized residuals from the mean-variance specification of each return series. For stationary return series, the autoregressive moving average (ARMA) models can be used to model the mean while the GARCH models can be used to capture the time-varying volatilities of each return series. To check whether there is a leverage effect on the volatilities of either series, a Threshold GARCH (TGARCH) model can be estimated. But first, stationarity of the return series should be tested using the Dickey-Fuller generalized least squares (ERS) test of unit root before proceeding to the joint estimation of the ARMA-GARCH models.

Two versions of the DCC model are used for this study. Under the assumption that the changes in the correlations are mean-reverting and temporary, the mean-reverting DCC model is employed. First, a matrix $Q_t = \Omega + \alpha\varepsilon_{t-1}\varepsilon'_{t-1} + \beta Q_{t-1}$, called the quasi-correlation matrix is postulated. To save up on the parameters in estimating the intercept matrix, correlation targeting is used to estimate the intercept parameters. The intercept estimate using correlation targeting is given by $\hat{\Omega} = (1-\alpha-\beta)\bar{R}$ where $\bar{R} = \frac{1}{T}\sum_{t=1}^{T}\varepsilon_t\varepsilon'_t$.

The new form of the mean-reverting DCC model is then given by:

$$Q_t = \bar{R} + \alpha\left(\varepsilon_{t-1}\varepsilon'_{t-1} - \bar{R}\right) + \beta\left(Q_{t-1} - \bar{R}\right) \tag{1}$$

It is guaranteed to be positive definite as long as the initial parameters α and β are all positive and that $\alpha + \beta < 1$.

The integrated DCC model on the other hand assumes that the quasi-correlation matrix has a unit root process that is, the process has no tendency to revert to an unconditional value and is useful in modelling correlations that have structural breaks and are unlikely to be reversed. The specification of the quasi-correlation process under the integrated DCC model is equal to $Q_t = \lambda\varepsilon_{t-1}\varepsilon'_{t-1} + (1-\lambda)Q_{t-1}$.

The dynamic parameters of both models are then estimated using maximum likelihood. Rescaling was also done in order to ensure that the quasi-correlation matrix is indeed a good approximation of the correlation matrix. This was done by both pre-multiplying and post-multiplying the square root of the diagonal elements of the quasi-correlation matrix with the quasi-correlation matrix. That is, the final estimated correlations are given by:

$$R_t = \text{diag}\{Q_t\}^{-\frac{1}{2}} Q_t \text{diag}\{Q_t\}^{-\frac{1}{2}} \quad (2)$$

In summary, the estimation of the parameters of the DCC models was done by utilizing a two-step estimator. Since the log-likelihood for multiple series can be expressed as the sum of the log-likelihood of the variance parameters and the log-likelihood of the correlation parameters, the first step of the estimation was done by maximizing the variance part of the likelihood function which is taken care of by the ARMA-GARCH estimation. The standardized residuals were then obtained from the first step and the log-likelihood of the correlation parameters was maximized by the estimation of the DCC specification.

4. Results and Discussion

4.1 Descriptive analysis

Figure 1 shows the levels of the Peso-Dollar exchange rate and the Philippine Stock Exchange index from the full sample of 3 January 2000 up to 26 February 2010. The composite index exhibits bullish periods between 2002 and 2007, and 2009 to 2010 and bearish periods on 2000 to 2002 and 2007 to 2009. For the earlier periods, the decline of the PSEi is subsequent to the effects of the tech bubble burst in the US. After 2002, a prolonged rebound can be seen as the PSEi rises until 2007 as the US housing bubble burst begins. The decline until 2008 shows its adverse effect to the local economy but 2009 shows signs of recovery as the PSEi levels begin to rise again. A strong evidence of a generally negative correlation is plausible as bullish stock market movements are generally accompanied by appreciative movements in the Peso with respect to the Dollar with the possible exception of the periods from early 2003 to 2004. Taking the viewpoint of Cappielo and De Santis' uncovered equity return parity condition, the co-movement of the Peso-Dollar exchange rate and PSE index from this period suggests that the Philippines had higher expected equity returns than that of the US during this time which might be coupled to a positive local economic outlook while the US is recovering from the 2000 crisis. Also, the last three quarters of 2007 show both series being positively correlated once again as they both drop steadily. This might be attributed to the adverse effect of the housing bubble burst in the US economy, which caused the depreciation of the Dollar, while the crisis affects the global equity markets.

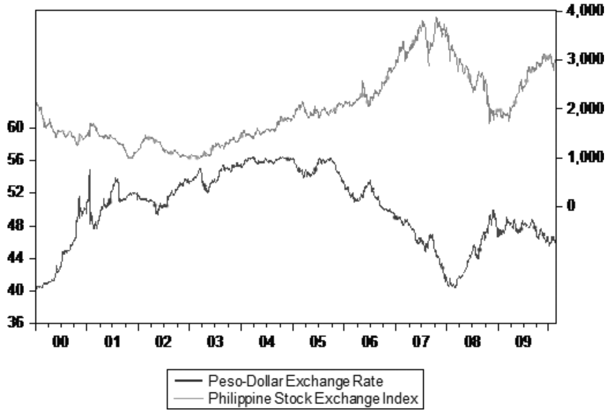

Figure 1. Time Plot of the Closing Daily Prices of the Philippine Stock Exchange Index and Peso-Dollar Exchange Rate from January, 2000 to February, 2010

Moreover, Figure 2 shows the time plot of the log returns for both assets by using the dlog[1] transformation. The returns of both assets appear to be stationary but volatility clustering and outliers are evident especially during the end of recessionary periods.

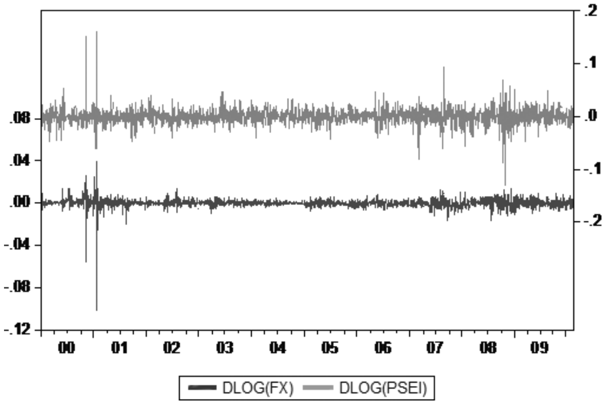

Figure 2. Time Plot of the Log Returns of the Peso-Dollar Exchange Rate and Philippine Stock Exchange Index from January, 2000 to February, 2010

1 $\text{dlog}(P_t) = \log(P_t) - \log(P_{t-1}) \approx (P_t - P_{t-1}) / P_{t-1}$, where P_t is the price of the asset at time t and log() is the natural

The sample correlation coefficients between the two daily log return series using moving windows of 22 days (i.e., 1 month), 60 days (i.e., 3 months), 125 days (i.e., 6 months), 250 days (i.e., 1 year), and 500 days (i.e., 2 years) are shown in Figure 3. The right and left axis of the graph show the short- (i.e., 1, 3, and 6 months) and medium-term (i.e., 1 and 2 years) moving correlations respectively. It is evident that the correlation changes over time and appears to be decreasing in the most recent periods as the economy recovers from the global financial crisis. The medium-term correlations are much smoother and easier to interpret over the short-term correlations. The monthly correlations have a great deal of volatility which looks like noise while the 3-month and 6-month correlations appear less fluctuating. However, the medium-term correlations do not capture the actual movements since the over-all pattern is most evident. There are no statistical criteria in choosing between these measures thus, in comparing the results of the correlation forecasting models, all of these five measures will be used as points of comparison.

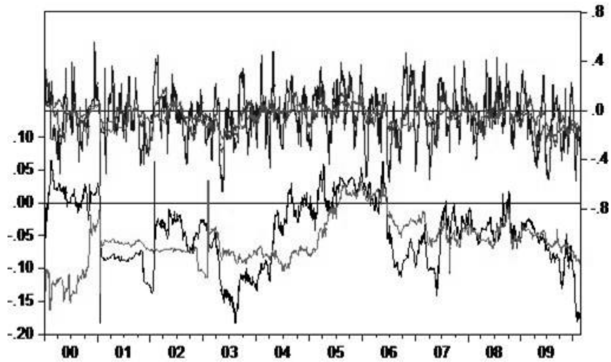

Figure 3. Time Plot of the 22-Day (1-Month), 60-Day (3-Month), 125-Day (6-Month), 250-Day (1-Year), and 500-Day (2-Year) Realized Correlations between the Log Returns of the Peso-Dollar Exchange Rate and Philippine Stock Exchange Index from January, 2000 to February, 2010

4.2 Univariate analysis

The model building process for the mean-variance specification of the Peso-Dollar exchange rate and PSE index log returns will only consider the in-sample observations as the estimation sample. The results of the ERS test for unit root for the log returns of the Peso-Dollar exchange rate and PSEi indicated that there is sufficient evidence to conclude stationarity for both returns. The test equations for the GLS detrended residuals of both log returns all have significant coefficients while the Elliott-Rothenberg-Stock DF-GLS test statistics (i.e., -9.34

and -36.82 for exchange rate and PSEi returns respectively) which indicated that the log returns for both series are stationary.[2] This result would enable the de-GARCHING process of the DCC model estimation to allow for the modelling of the mean and variance processes with common models such as the ARMA and GARCH respectively.

For the log returns of the Peso-Dollar exchange rate, an ARMA(1,3)-TGARCH(1,1) with conditional standardized Student-t (df=6.74) innovations was postulated while an AR(1)-TGARCH(1,1) with conditional standardized Student-t (df=5.12) innovations was jointly estimated as the final model for the log returns of the PSEi. Also for both returns, it is observed that the effect of negative returns on future volatility is higher compared to positive returns, a phenomenon known as a leverage effect.

4.3 In-sample performance

For a model with correlation targeting, the intercept of the quasi-correlation specification is a function of the average cross-products of the standardized residuals which is labelled as the R matrix and whose coefficients are shown in the table. The estimated alpha of 0.000386 (z-statistic=0.031, p=0.9755) is not statistically significant while the estimated beta of 0.854088 (z-statistic=17.498, p=0.0000) is statistically significant. Judging from the estimated mean-reverting DCC model, there seems to be sufficient evidence to say that the dynamic conditional correlation is not dependent on previous shocks but only to its previous value.

The initial values of alpha (0.05) and beta (0.86) where chosen since these values maximized the log-likelihood function. To see whether the estimation results are robust in terms of the initial values used, several runs of the maximum likelihood estimation were done while varying the initial values of alpha and beta and imposing the condition that their sum should be less than 1 to ensure positive definiteness of the resulting correlation matrices.

Unlike the mean-reverting DCC, the integrated DCC model has only one parameter to be estimated. The initial value of 0.01 for lambda of the maximum likelihood estimation was chosen in the same manner as that for the mean-reverting DCC. The log-likelihood achieves its highest value (-2318.879) for the smallest initial value of lambda (0.01) while the absolute log-likelihood exponentially increases as the initial value increases. The estimated lambda coefficient of 0.005542 (z-statistic=3.224, p=0.0013) indicates a substantial persistence of the correlations from the unconditional level.

The estimated in-sample correlations for both the mean-reverting and integrated DCC models were compared in Figure 4. The graph showed that the forecasts from the integrated DCC model are more persistent than those from

2 "covariance or weakly stationary"

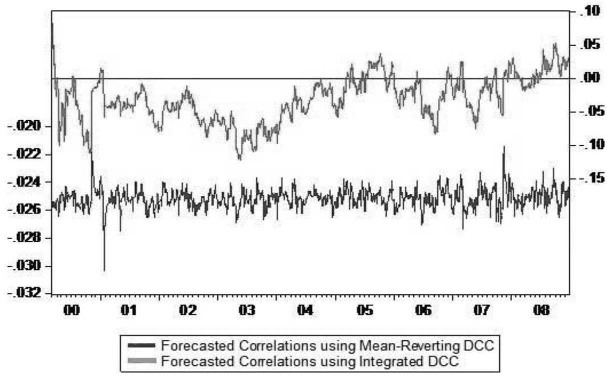

Figure 4. Time Plot of the Forecasted In-Sample Correlations between the Log Returns of the Peso-Dollar Exchange Rate and Philippine Stock Exchange Index from March 2000 to December 2008 using the Mean-Reverting and Integrated DCC Models

the mean-reverting DCC model. However, the range of fluctuations were less pronounced in the mean-reverting model since the forecasted correlations for the period shown were as low as approximately -0.03 and as high as -0.022 while the forecasts of the integrated DCC model ranged from -0.022 to as high as 0.08.

Figure 5 shows the correlation estimates from the multivariate GARCH models with those from the DCC models using the standardized residuals for both the log returns of the Peso-Dollar exchange rate and PSEi. The top part of the graph shows the forecasted correlations from the integrated DCC and diagonal VECH models which show fluctuations near the zero line, although the integrated DCC correlation forecasts are more persistent while the diagonal VECH forecasts are more volatile perhaps due to the non-positive definiteness of the forecasted correlation matrices inherent in the model itself. The bottom part of the graph shows the forecasts using the mean-reverting DCC, CCC, and diagonal BEKK models. The CCC fails to capture any dynamic pattern while being above the unconditional level of the diagonal BEKK and mean-reverting DCC estimates. The diagonal BEKK estimates are also volatile around the same level as the diagonal VECH estimates but have less extreme forecasts.

Forecast error measures such as the root mean squared error, mean absolute error, mean absolute percentage error, Theil inequality coefficient, and the bias, variance, and covariance proportions of the mean squared forecast error are computed for each model as compared to the different realized correlations to evaluate the performance of the models in estimating the correlations. It can be seen in Table 1 that the integrated DCC model yielded the lowest Root Mean Square

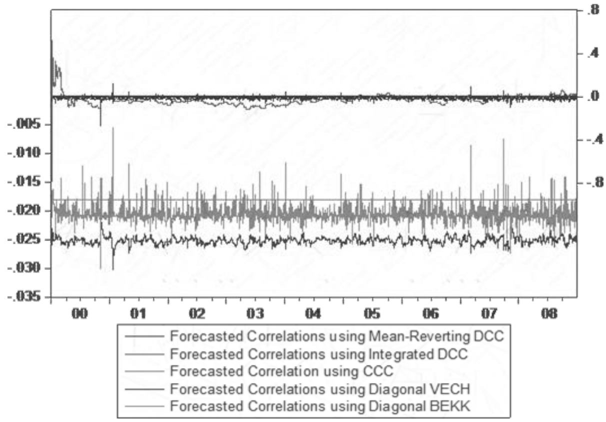

Figure 5. Time Plots of the Forecasted In-Sample Correlations between the Log Returns of the Peso-Dollar Exchange Rate and the Philippine Stock Exchange Index from March 2000 to December 2008 using the Mean-Reverting and Integrated Dynamic Conditional Correlation Models, Constant Conditional Correlation Model, Diagonal VECH Model, and Diagonal BEKK Model

Error (RMSE) for the short-term realized correlations while the mean-reverting DCC model is the lowest for the medium-term realized correlations. However, the integrated DCC model yielded the lowest Mean Absolute Error(MAE) for all the realized correlations. The integrated DCC model performed best in terms of RMSE for the short-term correlations but performed the worst for medium-term correlations. On the other hand, the integrated DCC model uniformly had the lowest MAE for all rolling window values. This might be attributed to the wild forecasts that the integrated DCC produced during the start-up months and that the RMSE exaggerates its effect since it is a squared error measure unlike the MAE which is an absolute error measure.

4.4 Out-of-sample performance

For the out-of-sample forecasting performance of the models, the observations from 5 January 2009 to 26 February 2010 were used and two types of forecasting were employed: rolling sample and dynamic forecasting.

4.4.1 Rolling sample forecasts

Among the models, the integrated DCC model generated a different forecast pattern from the 1-day, 2-day, 3-day, 4-day, and 5-day ahead correlation estimates which is generally declining while the multivariate GARCH models still fluctuate around an unconditional level.

The 1-day ahead forecasted correlations from all the models are then compared to the short-term realized correlations. As in the in-sample results, the

Table 1. In-sample Forecasting Performance of the Mean-Reverting DCC, Integrated DCC, CCC, Diagonal VECH, and Diagonal BEKK Models as Compared to the 22-Day, 60-Day, 125-Day, 250-Day, and 500-Day Realized Correlations

Forecast Sample: 1/04/2000 12/24/2008 Included Observations: 2212						
Correlation Proxy	Forecast Error Measures	MR-DCC	I-DCC	CCC	VECH	BEKK
	MAE	0.160148	0.152011	0.160524	0.162692	0.160644
	MAPE	128.5443	141.5271	118.1553	117.3225	121.3061
22-Day	THEIL	0.878250	0.729461	0.909462	0.913355	0.899608
	Biasprop	0.000457	0.000159	0.003205	0.004913	0.001965
	Varianceprop	0.996770	0.604763	0.996795	0.847256	0.980894
	Covarianceprop	0.002773	0.395077	2.30E-17	0.147831	0.017141
	RMSE	0.108278	0.092251	0.109325	0.111858	0.109122
	MAE	0.084624	0.071189	0.085203	0.087438	0.085197
	MAPE	218.0804	217.3490	180.4302	178.3159	195.5056
60-Day	THEIL	0.782000	0.536762	0.832032	0.838574	0.814929
	Biasprop	0.006200	0.005218	0.020325	0.027261	0.014485
	Varianceprop	0.986642	0.371916	0.979675	0.722696	0.956086
	Covarianceprop	0.007158	0.622866	4.25E-17	0.250043	0.029429
	RMSE	0.066365	0.061570	0.068138	0.070958	0.067564
	MAE	0.052208	0.044822	0.053464	0.055743	0.053085
	MAPE	1065.105	626.7145	778.9090	534.5831	858.9989
125-Day	THEIL	0.661295	0.460301	0.730397	0.744698	0.705284
	Biasprop	0.032560	0.026979	0.078042	0.095411	0.060236
	Varianceprop	0.952476	0.047905	0.921958	0.543435	0.894914
	Covarianceprop	0.014964	0.925116	6.68E-17	0.361154	0.044850
	RMSE	0.053166	0.054897	0.055938	0.059282	0.054985
	MAE	0.043066	0.038932	0.045066	0.047470	0.044400
	MAPE	301.6361	359.9563	237.5051	213.0582	259.5079
250-Day	THEIL	0.584788	0.441584	0.667137	0.690588	0.636730
	Biasprop	0.102570	0.076313	0.185422	0.206975	0.154811
	Varianceprop	0.878447	0.000710	0.814578	0.397373	0.793387
	Covarianceprop	0.018983	0.922977	7.61E-17	0.395652	0.051803
	RMSE	0.048983	0.066374	0.053985	0.057957	0.052282
	MAE	0.041791	0.040528	0.046760	0.049736	0.045054
	MAPE	100.0106	92.47362	97.30522	100.0270	98.35501
500-Day	THEIL	0.523890	0.523039	0.624615	0.655434	0.587845
	Biasprop	0.441525	0.213732	0.538272	0.537489	0.505043
	Varianceprop	0.540991	0.052896	0.461728	0.165353	0.452626
	Covarianceprop	0.017484	0.733373	5.87E-17	0.297158	0.042331

integrated DCC model is the only model that closely captured the trend of the declining realized short-term correlations though it does not capture its specific movements. The forecasts of other models were still relatively level and do not reflect the declining correlations.

To evaluate the performance of the models in estimating the correlations, forecast error measures as in the in-sample analysis were also computed for each model as compared to the different realized correlations and forecast horizons. For the 1-month and 2-month realized correlations, the integrated DCC has the lowest RMSE and MAE across all forecast horizons. On the other hand, the mean-reverting DCC has the lowest RMSE and MAE for the 2-year realized correlations. For the 6-month and 1-year realized correlations, the integrated DCC has the lowest RMSE while the mean-reverting DCC has the lowest MAE.

Several Diebold-Mariano tests using the squared error loss function were done to test whether the integrated DCC model significantly outperforms the other models in forecasting the realized correlations across the different rolling windows and forecast horizons. In Table 2, the Diebold-Mariano tests results show that there is enough evidence to say that the integrated DCC model has significantly greater predictive accuracy in forecasting the 3-month realized correlation across all forecast horizons and other models at 10% level of significance. However, the same conclusions cannot be said for the other realized correlations as proxies. The table also shows that the mean-reverting DCC has the same predictive accuracy across all forecast horizons than the integrated DCC if the 2-year realized correlations are used as proxy.

Based on the results, it can be concluded that the integrated DCC model is the most optimal model among the five correlation models in terms of short-run forecasts of the patterns of short-term realized correlations specifically the 3-month realized correlations of the log returns of the Peso-Dollar exchange rate and PSE index. This may be attributed to the fact that short-term correlations are generally volatile and persistent and the integrated DCC model performs well for persistent correlation series. However, for medium-term realized correlations of the log returns of the series, the mean-reverting DCC model seem to perform the best since these realized correlations are generally smoother and fluctuate around a fixed mean, and the mean-reverting DCC model performs well for correlations which revert to a constant value.

4.4.2 Dynamic forecasts

The forecasted correlations using dynamic forecasting are also compared with the short-term realized correlations in Figures 6. Because of the declining pattern of the realized correlations in the out-of-sample, all the models overestimated the correlation patterns but the mean-reverting DCC dynamic forecasts were the one closest to the average level of the realized forecasts.

Table 2. Diebold-Mariano Tests of Predictive Accuracy using the Squared Error Loss of the Integrated Dynamic Conditional Correlation Models versus the Mean-Reverting Dynamic Conditional Correlation Model, Constant Conditional Correlation Model, Diagonal VECH Model, and Diagonal BEKK Model using the 22-Day, 60-Day, 125-Day, 250-Day, and 500-Day Realized Correlations as Proxy for the True Correlation

Null Hypothesis: Integrated DCC has equal predictive accuracy as the other model Alternative Hypothesis: Integrated DCC has greater predictive accuracy than the other model Forecast Sample: 1/05/2009 2/26/2010 Method: Rolling Sample					
Correlation Proxy	Forecast Horizon	MR-DCC	CCC	VECH	BEKK
22-Day	1-Day	-1.68*	-2.35***	-3.30***	-2.32***
	2-Day	-1.58	-2.19**	-2.83***	-2.05**
	3-Day	-1.46	-2.04**	-2.72***	-1.90*
	4-Day	-1.34	-1.85*	-2.56***	-1.74*
	5-Day	-1.20	-1.66*	-2.35***	-1.56
60-Day	1-Day	-2.25**	-2.90***	-4.12***	-2.67***
	2-Day	-2.28**	-2.91***	-3.95***	-2.63***
	3-Day	-2.30**	-2.92***	-3.89***	-2.63***
	4-Day	-2.33***	-2.89***	-3.87***	-2.64***
	5-Day	-2.29**	-2.83***	-3.74***	-2.59***
125-Day	1-Day	-1.44	-2.06**	-3.17***	-1.86*
	2-Day	-1.41	-1.99**	-2.89***	-1.73*
	3-Day	-1.38	-1.94*	-2.80***	-1.68*
	4-Day	-1.35	-1.89*	-2.73***	-1.64
	5-Day	-1.30	-1.83*	-2.66***	-1.58
250-Day	1-Day	-0.83	-1.25	-2.29**	-1.11
	2-Day	-0.83	-1.23	-2.12**	-1.06
	3-Day	-0.83	-1.22	-2.06**	-1.05
	4-Day	-0.83	-1.20	-2.03**	-1.03
	5-Day	-0.82	-1.18	-1.99**	-1.00
500-Day	1-Day	0.77	0.26	-1.43	0.44
	2-Day	0.77	0.27	-1.17	0.50
	3-Day	0.77	0.29	-1.08	0.52
	4-Day	0.77	0.30	-1.02	0.53
	5-Day	0.78	0.32	-0.96	0.55
* Significant at 0.10 level ** Significant at 0.05 level*** Significant at 0.01 level					

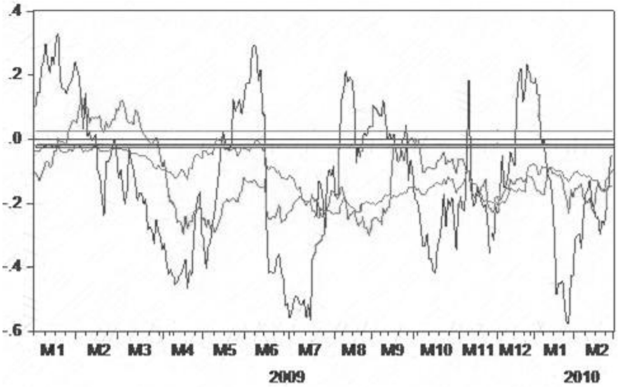

Figure 6. Time Plot of the Forecasted Dynamic Out-of-Sample Correlations between the Log Returns of the Peso-Dollar Exchange Rate and the Philippine Stock Exchange Index from January, 2009 to February, 2010 using the various forecasting models as compared to the Short-Term Realized Correlations

Table 3 shows the forecast error measures as also computed in the in-sample and rolling sample forecast analysis in each model as compared to the different realized correlations to evaluate the performance of the models in estimating the correlations. The statistics indicate that the mean-reverting DCC model has the lowest RMSE and MAE across all realized correlations in terms of its dynamic correlation forecasts.

These results are also evident in the plot of the rolling window used in the computation of the realized correlation versus the different forecast error measures of the dynamic forecasts for each model which is shown in Figure 7. Because the 1-day ahead forecast of the integrated DCC model is positive while the realized correlations for the out-of-sample periods were declining, it yielded the highest error statistics across all rolling window. However, the mean-reverting DCC performed well across all rolling windows, uniformly yielding the lowest RMSE and MAE.

Based on the results of the forecast error measures, there is evidence to say that the mean-reverting DCC model is the optimal model in dynamically forecasting the out-of-sample correlation level for the log returns of the Peso-Dollar exchange rate and PSE index.

Table 3. Dynamic Forecasting Performance of the Mean-Reverting DCC, Integrated DCC, CCC, Diagonal VECH, and Diagonal BEKK Models as Compared to the 22-Day, 60-Day, 125-Day, 250-Day, and 500-Day Realized Correlations

Forecast Sample: 1/04/2000 12/24/2008 Included Observations: 2212						
Correlation Proxy	Forecast Error Measures	MR-DCC	I-DCC	CCC	VECH	BEKK
22-Day	RMSE	0.245210	0.269657	0.248304	0.249640	0.247216
	MAE	0.205071	0.224459	0.207450	0.208499	0.206593
	MAPE	159.5745	163.6469	141.2357	133.9240	147.4181
	THEIL	0.869337	0.960974	0.902901	0.917384	0.891018
	Biasprop	0.181731	0.323352	0.201973	0.210290	0.194920
	Varianceprop	0.818212	0.676648	0.798027	0.787859	0.804889
	Covarianceprop	5.64E-05	NA	1.31E-17	0.001851	0.000191
60-Day	RMSE	0.144558	0.181763	0.149528	0.151617	0.147787
	MAE	0.127133	0.160191	0.131242	0.132964	0.129816
	MAPE	129.4439	149.8587	118.1320	113.7022	122.0401
	THEIL	0.768580	0.973934	0.825999	0.851108	0.805497
	Biasprop	0.478273	0.669987	0.512363	0.525582	0.500804
	Varianceprop	0.521628	0.330013	0.487637	0.472183	0.498956
	Covarianceprop	9.97E-05	NA	1.88E-17	0.002235	0.000240
125-Day	RMSE	0.108407	0.150095	0.114089	0.116473	0.112104
	MAE	0.087654	0.134803	0.093882	0.096601	0.091590
	MAPE	83.59381	152.2641	85.53014	87.08934	84.22336
	THEIL	0.702185	0.981481	0.774396	0.806367	0.748492
	Biasprop	0.629301	0.806608	0.665276	0.678546	0.653299
	Varianceprop	0.370607	0.193392	0.334724	0.318911	0.346423
	Covarianceprop	9.17E-05	NA	1.78E-17	0.002543	0.000278
250-Day	RMSE	0.064424	0.107321	0.070153	0.072576	0.068141
	MAE	0.051093	0.099878	0.058132	0.061019	0.055685
	MAPE	57.18574	140.4112	69.14957	74.05804	64.97343
	THEIL	0.581134	0.980982	0.675845	0.719174	0.641341
	Biasprop	0.628470	0.866103	0.686638	0.706867	0.667836
	Varianceprop	0.371362	0.133897	0.313362	0.289343	0.331726
	Covarianceprop	0.000168	NA	3.09E-17	0.003790	0.000438
500-Day	RMSE	0.037192	0.084340	0.043787	0.046532	0.041485
	MAE	0.034299	0.083104	0.041358	0.044245	0.038911
	MAPE	55.13589	142.2632	67.73576	72.87891	63.36843
	THEIL	0.431072	0.994338	0.552738	0.609584	0.507989
	Biasprop	0.850488	0.970915	0.892097	0.904104	0.879772
	Varianceprop	0.149334	0.029085	0.107903	0.092493	0.119796
	Covarianceprop	0.000177	NA	2.80E-17	0.003403	0.000432

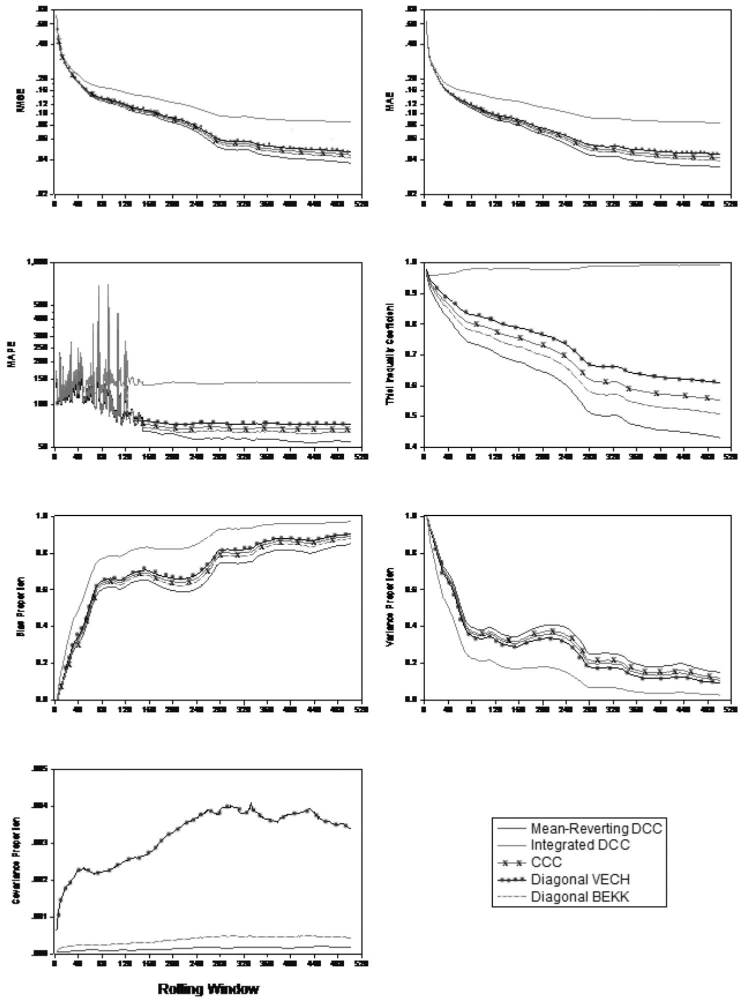

Figure 7. A Line Plot of the RMSE, MAE, MAPE, Theil Inequality Coefficient, and Bias, Variance, and Covariance Proportions of Mean Squared Forecast Error of the Forecasted Dynamic Out-of-Sample Correlations using the Mean-Reverting and Integrated Dynamic Conditional Correlation Models, Constant Conditional Correlation Model, Diagonal VECH Model, and Diagonal BEKK Model versus the Rolling Window of the Realized Correlation

5. Conclusions

An understanding in the nature and scope of correlations between the Peso-Dollar exchange rate and PSEi returns is important in the estimation of risks for optimal hedging since it does not involve only the quantification of individual volatilities but also include their pairwise correlations; as well as perceiving some ways to respond to the challenges in the economy. The dynamic conditional correlation model was used to estimate the correlation between these two returns. By employing both the mean-reverting and integrated DCC model, the results in this paper show that the correlation between the two returns is generally negative and is not really constant, but actually time-varying which is in accordance to the results of economic theories. The DCC models also produced more preferable results as to its flexibility, simplicity, and coherence with literature against other correlation models. Fluctuations in the correlations were caused by news affecting both the foreign exchange and equity markets. The integrated DCC model showed optimal forecast performance for the in-sample correlation patterns and rolling-sample forecasts of 1-day to 5-day horizons in terms of short-term realized correlations. On the other hand, the mean-reverting DCC model had the most desirable forecast properties for dynamic long-run forecasts and 1-day to 5-day rolling sample forecasts in terms of medium-term realized correlations. In terms of the short-run forecasting performance, the integrated DCC model was significantly higher in predictive accuracy than the other models with respect to the 3-month realized correlations.

REFERENCES

AQUINO, R. Q., 2005, Exchange rate risk and Philippine stock returns: Before and after the Asian financial crisis, *Applied Financial Economics* 15 (11), 765-771.

BAUTISTA, C. C., 2003, Interest rate-exchange rate dynamics in the Philippines: A DCC analysis, *Applied Economics Letters* 10, 107-111.

BENDEL, R. B., and AFIFI, A. A., 1977, Comparison of stopping rules in forward regression, *Journal of the American Statistical Association* 72, 46-53.

BOLLERSLEV, T., 1990, Modelling the coherence in short-run nominal exchange Rates: A Multivariate Generalized ARCH Model, *Review of Economics and Statistics* 69, 498-505.

BOLLERSLEV, T., ENGLE, R. F., and WOOLDRIDGE, J. M., 1988, A capital asset pricing model with time varying covariances, *Journal of Political Economy* 96, 116-31.

CAPPIELLO, L., and DE SANTIS, R. A., 2005, September, Explaining exchange rate dynamics: The uncovered equity return parity condition, European Central Bank Working Paper Series.

ENGLE, R., 2009, *Anticipating Correlations*, Princeton, New Jersey: Princeton University Press.

ENGLE, R. F., 2002, Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models, *Journal of Business and Economic Statistics* 20, 339-350.

HIGHLAND, L., and JOHNSON, M., 2004, *Landslide Types and Processes*. U.S. Geological Survey Fact Sheet.

PHILIPPINE NATIONAL RED CROSS, (2007, August 31, Retrieved October 18, 2010, from International Federation of Red Cross and Red Crescent Societies: http://www.ifrc.org/cgi/pdf_appeals.pl?06/MDRPH00107.pdf.

VARGAS, G. A., 2008, What drives the dynamic conditional correlation of foreign exchange and equity returns? *Munich Personal RePEc Archive*.

ACKNOWLEDGMENTS

The researchers would like to express their deepest gratitude to Gregorio A. Vargas of for sharing his EViews program codes for the estimation of the DCC models. Also, the researchers are grateful for Assistant Professor Peter Cayton from the University of the Philippines School of Statistics, and Lorelie Santos for their invaluable comments and suggestions regarding the study.

APPENDIX

Table A.1.Mean-Variance Process Estimation of the Log Returns of the Peso-Dollar Exchange Rate using the ARMA and TGARCH Model

Dependent Variable: DLOG(FX)
Method: ML - ARCH (Marquardt) - Student's t distribution
GARCH = C(5) + C(6)*RESID(-1)^2 + C(7)*RESID(-1)^2*(RESID(-1)<0)
+C(8)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
AR(1)	0.993983	0.004630	214.6750	0.0000
MA(1)	-0.817343	0.023453	-34.84953	0.0000
MA(2)	-0.253113	0.029550	-8.565532	0.0000
MA(3)	0.093909	0.022256	4.219585	0.0000
Variance Equation				
C	2.08E-07	5.21E-08	3.987031	0.0001
RESID(-1)^2	0.256945	0.031874	8.061387	0.0000
RESID(-1)^2*(RESID(-1)<0)	-0.072390	0.035929	-2.014805	0.0439
GARCH(-1)	0.786582	0.018408	42.73024	0.0000

Table A.2. Mean-Variance Process Estimation of the Log Returns of the Philippine Stock Exchange Index using the ARMA and TGARCH Model

Dependent Variable: DLOG(PSEI)
Method: ML - ARCH (Marquardt) - Student's t distribution
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*RESID(-1)^2*(RESID(-1)<0) + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
AR(1)	0.121143	0.022085	5.485206	0.0000
Variance Equation				
C	2.07E-05	4.44E-06	4.671300	0.0000
RESID(-1)^2	0.093854	0.026344	3.562687	0.0004
RESID(-1)^2*(RESID(-1)<0)	0.115350	0.038224	3.017737	0.0025
GARCH(-1)	0.745215	0.037805	19.71231	0.0000

Table B.1. Estimated Parameters of the Mean-Reverting DCC Model between the Log Returns of Peso-Dollar Exchange Rate and the Log Returns of Philippine Stock Exchange Index with an ARMA-TGARCH Specification

	Coefficient	Std. Error	z-Statistic	Prob.
R(1,1)	1.012139	--	--	--
R(1,2)	-0.026112	--	--	--
R(2,2)	1.066428	--	--	--
ALPHA(1)	0.000386	0.012565	0.030715	0.9755
BETA(1)	0.854088	0.048812	17.49752	0.0000
Log likelihood	-2299.238	Akaike info criterion		2.080686
Avg. log likelihood	-1.039439	Schwarz criterion		2.085841
Number of Coefs.	2	Hannan-Quinn criter.		2.082569
Minimum Eigenvalue	0.145845			

Table B.2. Estimated Parameters of the Integrated DCC Model between the Log Returns of the Peso-Dollar Exchange Rate and the Log Returns of Philippine Stock Exchange Index with an ARMA-TGARCH Specification

	Coefficient	Std. Error	z-Statistic	Prob.
LAMBDA(1)	0.005542	0.001719	3.224459	0.0013
Log likelihood	-2318.879	Akaike info criterion	2.097539	2.080686
Avg. log likelihood	-1.048318	Schwarz criterion	2.100117	2.085841
Number of Coefs.	1	Hannan-Quinn criter.	2.098481	2.082569
Minimum Eigenvalue	0.000409			

Table C.1. Multivariate GARCH Estimation of the Standardized Residuals of the Log Returns of the Peso-Dollar Exchange Rate and Philippine Stock Exchange Index using the Constant Conditional Correlation Model

t-Distribution (Degree of Freedom)				
C(8)	6.666881	0.510534	13.05863	0.0000
Log likelihood	-6151.564	Schwarz criterion	5.587322	
Avg. log likelihood	-1.389870	Hannan-Quinn criter.	5.574239	
Akaike info criterion	5.566709			
Covariance specification: Constant Conditional Correlation GARCH(i) = M(i) + A1(i)*RESID(i)(-1)^2 + B1(i)*GARCH(i)(-1) COV(i,j) = R(i,j)*@SQRT(GARCH(i)*GARCH(j))				
Transformed Variance Coefficients				
	Coefficient	**Std. Error**	**z-Statistic**	**Prob.**
M(1)	1.121890	0.513654	2.184137	0.0290
A1(1)	0.036470	0.022485	1.622003	0.1048
B1(1)	-0.124713	0.497366	-0.250748	0.8020
M(2)	0.074407	0.041414	1.796659	0.0724
A1(2)	-0.004500	0.000294	-15.31137	0.0000
B1(2)	0.928468	0.042589	21.80067	0.0000
R(1,2)	-0.018073	0.025527	-0.707989	0.4790

Table C.2. Multivariate GARCH Estimation of the Standardized Residuals of the Log Returns of the Peso-Dollar Exchange Rate and Philippine Stock Exchange Index using the Diagonal VECH Model

t-Distribution (Degree of Freedom)				
C(10)	6.665167	0.508201	13.11521	0.0000
Log likelihood	-6151.541	Schwarz criterion	5.594262	
Avg. log likelihood	-1.389865	Hannan-Quinn criter.	5.577908	
Akaike info criterion	5.568496			
Covariance specification: Diagonal VECH GARCH = M + A1.*RESID(-1)*RESID(-1)' + B1.*GARCH(-1) M is an indefinite matrix, A1 is an indefinite matrix*, B1 is an indefinite matrix*				
Transformed Variance Coefficients				
	Coefficient	**Std. Error**	**z-Statistic**	**Prob.**
M(1,1)	1.120896	0.525604	2.132589	0.0330
M(1,2)	-0.014730	0.040507	-0.363638	0.7161
M(2,2)	0.056941	0.034528	1.649105	0.0991
A1(1,1)	0.036215	0.022590	1.603152	0.1089
A1(1,2)	-0.012553	0.016173	-0.776174	0.4376
A1(2,2)	-0.004000	0.001030	-3.883206	0.0001
B1(1,1)	-0.123015	0.509672	-0.241361	0.8093
B1(1,2)	0.045083	2.011982	0.022407	0.9821
B1(2,2)	0.945772	0.035088	26.95392	0.0000
* Coefficient matrix is not PSD.				

Table C.3. Multivariate GARCH Estimation of the Standardized Residuals of the Log Returns of the Peso-Dollar Exchange Rate and Philippine Stock Exchange Index using the Diagonal BEKK Model

t-Distribution (Degree of Freedom)				
C(8)	6.647024	0.498373	13.33744	0.0000
Log likelihood	-6156.005	Schwarz criterion	5.591336	
Avg. log likelihood	-1.390873	Hannan-Quinn criter.	5.578253	
Akaike info criterion	5.570723			
Covariance specification: Diagonal BEKK GARCH = M + A1*RESID(-1)*RESID(-1)'*A1 + B1*GARCH(-1)*B1 M is an indefinite matrix, A1 is a diagonal matrix, B1 is a diagonal matrix				
Transformed Variance Coefficients				
	Coefficient	Std. Error	z-Statistic	Prob.
M(1,1)	0.993584	0.371752	2.672705	0.0075
M(1,2)	-0.020359	0.314464	-0.064742	0.9484
M(2,2)	0.064023	0.161543	0.396322	0.6919
A1(1,1)	0.192825	0.058583	3.291464	0.0010
A1(2,2)	-0.007555	0.113330	-0.066664	0.9468
B1(1,1)	0.011562	15.69921	0.000736	0.9994
B1(2,2)	0.966503	0.085785	11.26662	0.0000