Classes of Estimators under New Calibration Schemes using Non-conventional Measures of Dispersion

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In this paper, we proposed two classes of estimators under two new calibration schemes for a heterogeneous population by incorporating auxiliary information of Non-Conventional Measures of dispersion which are robust against the presence of outlier in the data.Theoretical results are supported by simulation studies conducted on six bivariate populations generated using exponential and normal distributions. The biases and percentage relative efficiencies (PRE) of the proposed and other related estimators in the study were computed and results indicated that the estimators proposed under suggested calibration schemes perform on average more efficiently than conventional unbiased estimator, Rao and Khan (2016) and Nidhi et al. (2017).

Keywords: *heterogeneous population, Outliers, Estimators, Robust measures, Population mean*

1. Introduction

Traditional method of estimating mean of a study variable y in heterogeneous population stratified into K homogeneous non-overlapping subgroups is to use conventional estimator defined in Eq. (1) as follow:

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$$\tau_{st} = \sum_{h=1}^{K} \Psi_h \overline{y}_h \tag{1}$$

where, $\Psi_h = N_h / N$, $\overline{y}_h = n_h^{-1} \sum_{i=1}^{n_n} y_{hi}$, n_h is sample size of units drawn with SRSWOR from stratum h, N_h is the size of stratum h and y_{hi} is the i^{th} observation of stratum h.

Utilizing information on supplementary variables to improve the precision of estimators at planning, designing and estimation stage is a well-known approach in sampling theory. Estimation, especially in stratified sampling, entails attaching weight to sample data followed by calculating the weighted mean. Deville and Sarndal (1992) suggested modified weights which improve the precision of an estimate using a procedure called calibration. Many authors have proposed estimators and studied their properties in this direction including Singh & Mohl (1996), Estevao and Sarndal (2000), Audu et al. (2020a), Audu et al. (2020b) and Audu et al. (2021). Tracy et al. (2003) obtained calibration weights for population mean by using first and second order moment of auxiliary variable in stratified random sampling. Kim et al. (2007) utilized calibration approach in defining estimators for population variance in stratified random sampling. Barktus and Pumputis (2010) proposed calibration estimator in stratified sampling for estimating population ratio. Sud et al. (2014) and Estevao & Sarndal (2002) have proposed estimators for different population parameters under different sampling schemes that satisfy the underlying constraints. The weights in stratified sampling are only a function of stratum size which does not gives importance to the stratum information.

Rao and Khan (2016) suggested two new calibration schemes by additively transforming both stratum sample and population means of auxiliary variable using sample and population coefficient of variation respectively in the constraints with respect to usual unbiased estimator $\tau_0 = \sum_{h=1}^{K} \Psi_h \overline{y}_h$, where $\Psi_h = N_h / N$ is the stratum weight and $\overline{y}_h$ is the stratum average of study variable y. The calibration weights Ψ^*_{h1} and Ψ^*_{h2} are selected so as to minimize the distance function $Z_j = \sum_{h=1}^{K} \left(\Psi^*_{hj} - \Psi_h\right)^2 / \Psi_h \phi_h,\ j = 1, 2$ subject to calibration constraints $\sum_{h=1}^{K} \Psi^*_{h1} \left(\overline{x}_h + c_{xh}\right) = \sum_{h=1}^{K} \Psi_h \left(\overline{X}_h + C_{Xh}\right)$ and $\sum_{h=1}^{K} \Psi^*_{h2} \left(\overline{x}_h + c_{xh} + 1\right) = \sum_{h=1}^{K} \Psi_h \left(\overline{X}_h + C_{Xh} + 1\right)$ respectively, where $\overline{x}_h$ and $\overline{X}_h$ are sample mean and population mean of h^{th} stratum

$$c_{xh}=\frac{s_{xh}}{\bar{x}_h}, C_{Xh}=\frac{S_{Xh}}{\bar{X}_h},, s_{xh}^2=\frac{\sum_{i=1}^{n_h}\left(x_{hi}-\bar{x}_h\right)^2}{n_h-1}, \bar{x}_h=\frac{1}{n_h}\sum_{i=1}^{n_h}x_{hi}, S_{Xh}^2=\frac{\sum_{i=1}^{N_h}\left(x_{hi}-\bar{X}_h\right)^2}{N_h-1}$$

The two schemes proposed are as follow;

$$\tau_{RK1}=\sum_{h=1}^{K}\Psi_h\bar{y}_h\sum_{h=1}^{K}\Psi_h\left(\bar{X}_h+C_X\right)\left(\sum_{h=1}^{K}\Psi_h\left(\bar{x}_h+c_x\right)\right)^{-1} \tag{2}$$

$$\tau_{RK2}=\sum_{h=1}^{K}\Psi_h\bar{y}_h\sum_{h=1}^{K}\Psi_h\left(1+\bar{X}_h+C_X\right)\left(\sum_{h=1}^{K}\Psi_h\left(1+\bar{x}_h+c_x\right)\right)^{-1} \tag{3}$$

where Ψ_h is the stratum weight, C_x is the population coefficient of variation of X, and c_x is the sample coefficient of variation of X.

However, τ_{RK1} and τ_{RK2} are functions of coefficients of variation which can be affected by the presence of extreme values or outliers.

Recently, Nidhi et al. (2017) suggested a new calibration procedure with respect to usual unbiased estimator $\tau_0=\sum_{h=1}^{K}\Psi_h\bar{y}_h$, where $\Psi_h = N_h/N$ is the stratum weight, and $\bar{y}_h$ is the stratum average of study variable y. The calibration weights Ψ_h^* is selected so as to minimize the distance function $Z=\sum_{h=1}^{K}\left(\Psi_h^*-\Psi_h\right)^2/\Psi_h\phi_h$ subject to two calibration constraints $\sum_{h=1}^{K}\Psi_h^*\bar{x}_h=\sum_{h=1}^{K}\Psi_h\bar{X}_h$ and $\sum_{h=1}^{K}\Psi_h^*=1$, where $\bar{x}_h$ and $\bar{X}_h$ are sample mean and population mean of h^{th} stratum. For the cases $\varphi_h=1$ and $\phi_h=\bar{x}_h^{-1}$, Nidhi et al. (2017) obtained new calibrated estimators

$$\tau_{NSSS1}=\sum_{h=1}^{K}\Psi_h\bar{y}_h+\hat{\beta}_{st1}\left(\bar{X}-\sum_{h=1}^{K}\Psi_h\bar{x}_h\right) \tag{4}$$

and

$$\tau_{NSSS2}=\sum_{h=1}^{K}\Psi_h\bar{y}_h+\hat{\beta}_{st2}\left(\bar{X}-\sum_{h=1}^{K}\Psi_h\bar{x}_h\right) \tag{5}$$

respectively where

$$\hat{\beta}_{st1}=\frac{\sum_{h=1}^{K}\Psi_h\bar{x}_h\bar{y}_h-\sum_{h=1}^{K}\Psi_h\bar{y}_h\sum_{h=1}^{K}\Psi_h\bar{x}_h}{\sum_{h=1}^{K}\Psi_h\bar{x}_h^2-\left(\sum_{h=1}^{K}\Psi_h\bar{x}_h\right)^2}$$

and

$$\hat{\beta}_{st2} = \frac{\sum_{h=1}^{K}\Psi_h\bar{y}_h\sum_{h=1}^{K}\Psi_h/\bar{x}_n - \sum_{h=1}^{K}\Psi_h\bar{y}_h/\bar{x}_h}{\sum_{h=1}^{K}\Psi_h\bar{x}_h\sum_{h=1}^{K}\Psi_h/\bar{x}_n - 1}$$

.

2. New Calibration Estimators

The coefficient of variation is affected by outliers, hence, an alternative to the estimators τ_{RK1} and τ_{RK2} would be to replace the coefficient of variation with robust measures of dispersion. Measures of dispersion which are robust to outliers are useful in cases when the population departs from normality. Motivated by Subzar et al. (2018), we proposed new calibration estimators in stratified random sampling using information on robust measures such as Gini's mean difference $G_M(g_M)$, Downton's method $D_M(d_M)$ and probability weighted moments $P_M(p_M)$.

Let $z \in \Re^+$ be population with units z_i, 1,2,…, N, then;

$$G_M(z) = 2N^{-1}(N-1)^{-1}\sum_{i=1}^{N}(2i-N-1)z_i \tag{6}$$

$$D_M(z) = 2\sqrt{\pi}N^{-1}(N-1)^{-1}\sum_{i=1}^{N}(i-(N+1)/2)z_i \tag{7}$$

$$P_{WM}(z) = \sqrt{\pi}N^{-2}\sum_{i=1}^{N}(2i-(N+1))z_i \tag{8}$$

Also, let u be sample with unit u_i, 1,2,…, n, , then;

$$g_M(u) = 2n^{-1}(n-1)^{-1}\sum_{i=1}^{n}(2i-n-1)u_i \tag{9}$$

$$d_M(u) = 2\sqrt{\pi}n^{-1}(n-1)^{-1}\sum_{i=1}^{n}(i-(n+1)/2)u_i \tag{10}$$

$$p_M(u) = \sqrt{\pi}n^{-2}\sum_{i=1}^{n}(2i-(n+1))u_i \tag{11}$$

Downton's Method, Gini's Mean Method and Probability Weighted Method have been studied by several authors (see David 1968, Downton 1966, Greenwood et al 1979, Yitzhaki 2003). Some existing literature on the improvement of estimators that utilized these robust functions include Abid et al. (2016), Gupta and Yadav (2017) and Yadav and Zaman (2021).

2.1. First new calibration scheme

To obtain the first class of calibration estimator, consider estimator defined in Eq. (12) in stratified sampling;

$$\tau_{ARi} = \sum_{h=1}^{K}\Theta_{hi}^{*}\bar{y}_h,\;\; i = 1,2,3. \tag{12}$$

where Θ^*_{hi} is the new calibration weight that minimizes the Chi-square function denoted Z^* subject to constraints involving the non-standard measures of dispersion, that is,

$$\left.\begin{aligned} &\min \quad Z^* = \sum\nolimits_{h=1}^{K}\left(\Theta^*_{hi} - \Psi_h\right)^2 / \Psi_h \phi_h \\ &s.t \sum\nolimits_{h=1}^{K} \Theta^*_{hi}\left(\bar{x}_h + v_{ih}(x)\right) = \sum_{h=1}^{K} \Psi_h \left(\bar{X}_h + V_{ih}(x)\right) \\ &\quad \sum\nolimits_{h=1}^{K} \Theta^*_{hi} = 1 \end{aligned}\right\} \tag{13}$$

where $\phi_h > 0$ in (13) are suitably chosen weights which determine the form of estimator,

$$V_{1h}(x) = G_{Mh}(x), V_{2h}(x) = D_{Mh}(x),\ V_{3h}(x) = P_{Mh}(x),$$

$$v_{1h}(x) = g_{Mh}(x), v_{2h}(x) = d_{Mh}(x),\ v_{3h}(x) = p_{Mh}(x)$$

This minimization problem may be solved by the method of Lagrange multipliers.

Consider the following function

$$L_g = \sum_{h=1}^{K} \frac{\left(\Theta^*_{hi} - \Psi_h\right)^2}{\Psi_h \phi_h} - 2\lambda_1 \left(\sum_{h=1}^{K} \Theta^*_{hi}\left(\bar{x}_h + v_{ih}(x)\right) - \sum_{h=1}^{K} \Psi_h \left(\bar{X}_h + V_{ih}(x)\right) \right) \tag{14}$$
$$- 2\lambda_2 \left(\sum\nolimits_{h=1}^{K} \Theta^*_{hi} - 1 \right)$$

where λ_j, j =1,2 is Lagrange multiplier. Then, differentiate Lg with respect to $\Theta^*_{hi}, \lambda_1, \lambda_2,$ and equate to 0, that is,

$$\frac{\partial L_g}{\partial \Theta^*_{hi}} = 0,\ \frac{\partial L_g}{\partial \lambda_1} = 0, \frac{\partial L_g}{\partial \lambda_2} = 0 \tag{15}$$

Solving Eq.(15), we get Eq. (16), Eq.(17) and Eq.(18);

$$\Theta^*_{hi} = \Psi_h + \lambda_1 \Psi_h \phi_h \left(\bar{x}_h + v_{ih}(x)\right) + \lambda_2 \Psi_h \phi_h \tag{16}$$

$$\sum\nolimits_{h=1}^{K} \Theta^*_{hi}\left(\bar{x}_h + v_{ih}(x)\right) - \sum\nolimits_{h=1}^{K} \Psi_h \left(\bar{X}_h + V_{ih}(x)\right) = 0 \tag{17}$$

$$\sum\nolimits_{h=1}^{K} \Theta^*_{hi} - 1 = 0 \tag{18}$$

Substituting the value obtained from Eq. (16) in Eq. (17) and Eq. (18), we get Eq. (19) and Eq. (20) as;

$$\lambda_1 \sum_{h=1}^{K} \Psi_h \phi_h \left(\bar{x}_h + \nu_{hi}(x)\right)^2 + \lambda_2 \sum_{h=1}^{K} \Psi_h \phi_h \left(\bar{x}_h + \nu_{hi}(x)\right) = \sum_{h=1}^{K} \Psi_h \left(\bar{X}_h + V_{hi}(x)\right) - \sum_{h=1}^{K} \Psi_h \left(\bar{x}_h + \nu_{hi}(x)\right) \tag{19}$$

$$\lambda_1 \sum_{h=1}^{K} \Psi_h \phi_h \left(\bar{x}_h + \nu_{hi}(x)\right) + \lambda_2 \sum_{h=1}^{K} \Psi_h \phi_h = 0 \tag{20}$$

Solving Eq. (19) and Eq. (20) simultaneously, we get expression for λ_1 and λ_2 denoted by λ_1^{opt} and λ_2^{opt} respectively as;

$$\lambda_1^{opt} = \frac{\sum_{h=1}^{K} \Psi_h \phi_h \left(\sum_{h=1}^{K} \Psi_h \left(\bar{X}_h + V_{hi}(x)\right) - \sum_{h=1}^{K} \Psi_h \left(\bar{x}_h + \nu_{hi}(x)\right)\right)}{\sum_{h=1}^{K} \Psi_h \phi_h \sum_{h=1}^{K} \Psi_h \phi_h \left(\bar{x}_h + \nu_{hi}(x)\right)^2 - \left(\sum_{h=1}^{K} \Psi_h \phi_h \left(\bar{x}_h + \nu_{hi}(x)\right)\right)^2} \tag{21}$$

$$\lambda_2^{opt} = -\frac{\sum_{h=1}^{K} \Psi_h \phi_h \left(\bar{x}_h + \nu_{hi}(x)\right)\left(\sum_{h=1}^{K} \Psi_h \left(\bar{X}_h + V_{hi}(x)\right) - \sum_{h=1}^{K} \Psi_h \left(\bar{x}_h + \nu_{hi}(x)\right)\right)}{\sum_{h=1}^{K} \Psi_h \phi_h \sum_{h=1}^{K} \Psi_h \phi_h \left(\bar{x}_h + \nu_{hi}(x)\right)^2 - \left(\sum_{h=1}^{K} \Psi_h \phi_h \left(\bar{x}_h + \nu_{hi}(x)\right)\right)^2} \tag{22}$$

Now, substituting Eq.(21) and Eq.(22) in Eq.(16), the new calibrated weights Θ_{hi}^* are obtained as

$$\Theta_{hi}^* = \Psi_h + \lambda_1^{opt} \Psi_h \phi_h \left(\bar{x}_h + \nu_{hi}(x)\right) + \lambda_2^{opt} \Psi_h \phi_h \tag{23}$$

and the new class of calibrated estimators is obtained as;

$$\tau_{ARi} = \sum_{h=1}^{K} \Psi_h \bar{y}_h + \rho_{st}^* \left(\sum_{h=1}^{K} \Psi_h \left(\bar{X}_h + V_{hi}(x)\right) - \sum_{h=1}^{K} \Psi_h \left(\bar{x}_h + \nu_{hi}(x)\right)\right), \quad i = 1, 2, 3 \tag{24}$$

where

$$\rho_{st}^* = \frac{\sum_{h=1}^{K} \Psi_h \phi_h \sum_{h=1}^{K} \Psi_h \phi_h \left(\bar{x}_h + \nu_{hi}(x)\right) \bar{y}_h - \sum_{h=1}^{K} \Psi_h \phi_h \bar{y}_h \sum_{h=1}^{K} \Psi_h \phi_h \left(\bar{x}_h + \nu_{hi}(x)\right)}{\sum_{h=1}^{K} \Psi_h \phi_h \sum_{h=1}^{K} \Psi_h \phi_h \left(\bar{x}_h + \nu_{hi}(x)\right)^2 - \left(\sum_{h=1}^{K} \Psi_h \phi_h \left(\bar{x}_h + \nu_{hi}(x)\right)\right)^2}$$

This estimator has estimated mean squared error (MSE) denoted by $M\hat{S}E\left(\tau_{ARi}\right)$ given by;

$$M\hat{S}E\left(\tau_{ARi}\right)=\mathrm{v}\left(\bar{y}_{st}\right)+\rho_{st}^{*^2}\,\mathrm{v}\left(\bar{x}_{st}\right)-2\rho_{st}^{*}\,\mathrm{cov}\left(\bar{y}_{st}\bar{x}_{st}\right) \tag{25}$$

where

$$\begin{aligned}
v\left(\bar{y}_{st}\right)&=\sum\nolimits_{h=1}^{K}\Psi_h\gamma_h S_{yh}^2, v\left(\bar{x}_{st}\right)\\
&=\sum\nolimits_{h=1}^{K}\Psi_h\gamma_h S_{xh}^2, \mathrm{cov}\left(\bar{y}_{st}\bar{x}_{st}\right)\\
&=\sum\nolimits_{h=1}^{K}\Psi_h\gamma_h\rho_{yxh}S_{yh}S_{xh}, \gamma_h=\frac{1}{n_h}-\frac{1}{N_h}
\end{aligned}$$

Further, substituting $\phi_h=\left(\bar{x}_h+\nu_{ih}(x)\right)^{-1}$, and $\nu_{hi}(x)$ be either $g_{Mh}(x)$ or $d_{Mh}(x)$ or $p_{Mh}(x)$ we obtained new estimators as;

$$\left.\begin{aligned}
\tau_{AR1}&=\sum\nolimits_{h=1}^{K}\Psi_h\bar{y}_h+\rho_{st1}^{*}\left(\sum\nolimits_{h=1}^{K}\Psi_h\left(\bar{X}_h+G_{Mh}(x)\right)-\sum\nolimits_{h=1}^{K}\Psi_h\left(\bar{x}_h+g_{Mh}(x)\right)\right)\\
\tau_{AR2}&=\sum\nolimits_{h=1}^{K}\Psi_h\bar{y}_h+\rho_{st2}^{*}\left(\sum\nolimits_{h=1}^{K}\Psi_h\left(\bar{X}_h+D_{Mh}(x)\right)-\sum\nolimits_{h=1}^{K}\Psi_h\left(\bar{x}_h+d_{Mh}(x)\right)\right)\\
\tau_{AR3}&=\sum\nolimits_{h=1}^{K}\Psi_h\bar{y}_h+\rho_{st3}^{*}\left(\sum\nolimits_{h=1}^{K}\Psi_h\left(\bar{X}_h+P_{Mh}(x)\right)-\sum\nolimits_{h=1}^{K}\Psi_h\left(\bar{x}_h+p_{Mh}(x)\right)\right)
\end{aligned}\right\} \tag{26}$$

where

$$\rho_{sti}^{*}=\frac{\sum\nolimits_{h=1}^{K}\Psi_h\left(\bar{x}_h+\nu_{hi}(x)\right)^{-1}\sum\nolimits_{h=1}^{K}\Psi_h\bar{y}_h-\sum\nolimits_{h=1}^{K}\Psi_h\left(\bar{x}_h+\nu_{hi}(x)\right)^{-1}\bar{y}_h}{\sum\nolimits_{h=1}^{K}\Psi_h\left(\bar{x}_h+\nu_{hi}(x)\right)^{-1}\sum\nolimits_{h=1}^{K}\Psi_h\left(\bar{x}_h+\nu_{hi}(x)\right)-1},$$

$i=1,2,3$

2.2. Second new calibration scheme

To obtain the second class of the proposed estimators, we let

$$\tau_{ASi}=\sum_{h=1}^{K}\mathrm{H}_{hi}^{*}\bar{y}_h,\quad i=1,2,3. \tag{27}$$

where H_{hi}^{*} is the new calibration weight such that the Chi-square function T^{*} is defined as

$$\left.\begin{aligned} &\min \quad T^* = \sum_{h=1}^{K}\left(\mathrm{H}_{ih}^* - \Psi_h\right)^2 / \Psi_h \phi_h \\ &s.t \sum_{h=1}^{K}\mathrm{H}_{ih}^*\left(1+\bar{x}_h + \nu_{ih}(x)\right) = \sum_{h=1}^{K}\Psi_h\left(1+\bar{X}_h + V_{ih}(x)\right) \\ &\quad \sum_{h=1}^{K}\mathrm{H}_{ih}^* = 1 \end{aligned}\right\} \tag{28}$$

Solving for new calibrated weights H_{hi}^* using the Lagrange multipliers technique, the new calibrated weights H_{hi}^* is

$$\mathrm{H}_{hi}^* = \Psi_h + \mu_1^{opt}\Psi_h\phi_h\left(1+\bar{x}_h + \nu_{hi}(x)\right) + \mu_2^{opt}\Psi_h\phi_h, \tag{29}$$

where

$$\mu_1^{opt} = \frac{\sum_{h=1}^{K}\Psi_h\phi_h\left(\sum_{h=1}^{K}\Psi_h\left(1+\bar{X}_h + V_{hi}(x)\right) - \sum_{h=1}^{K}\Psi_h\left(1+\bar{x}_h+\nu_{hi}(x)\right)\right)}{\sum_{h=1}^{K}\Psi_h\phi_h\sum_{h=1}^{K}\Psi_h\phi_h\left(1+\bar{x}_h+\nu_{hi}(x)\right)^2 - \left(\sum_{h=1}^{K}\Psi_h\phi_h\left(1+\bar{x}_h+\nu_{hi}(x)\right)\right)^2},$$

$$\mu_2^{opt} = -\frac{\sum_{h=1}^{K}\Psi_h\phi_h\left(1+\bar{x}_h+\nu_{hi}(x)\right)\left(\sum_{h=1}^{K}\Psi_h\left(1+\bar{X}_h + V_{hi}(x)\right) - \sum_{h=1}^{K}\Psi_h\left(1+\bar{x}_h+\nu_{hi}(x)\right)\right)}{\sum_{h=1}^{K}\Psi_h\phi_h\sum_{h=1}^{K}\Psi_h\phi_h\left(1+\bar{x}_h+\nu_{hi}(x)\right)^2 - \left(\sum_{h=1}^{K}\Psi_h\phi_h\left(1+\bar{x}_h+\nu_{hi}(x)\right)\right)^2}$$

and the new class of calibrated estimators is obtained as:

$$\tau_{ASi} = \sum_{h=1}^{K}\Psi_h\bar{y}_h + \sigma_{st}^*\left(\sum_{h=1}^{K}\Psi_h\left(1+\bar{X}_h + V_{hi}(x)\right) - \sum_{h=1}^{K}\Psi_h\left(1+\bar{x}_h+\nu_{hi}(x)\right)\right), \quad i = 1,2,3 \tag{30}$$

where

$$\sigma_{st}^* = \frac{\sum_{h=1}^{K}\Psi_h\phi_h\sum_{h=1}^{K}\Psi_h\phi_h\left(1+\bar{x}_h+\nu_{hi}(x)\right)\bar{y}_h - \sum_{h=1}^{K}\Psi_h\phi_h\bar{y}_h\sum_{h=1}^{K}\Psi_h\phi_h\left(1+\bar{x}_h+\nu_{hi}(x)\right)}{\sum_{h=1}^{K}\Psi_h\phi_h\sum_{h=1}^{K}\Psi_h\phi_h\left(1+\bar{x}_h+\nu_{hi}(x)\right)^2 - \left(\sum_{h=1}^{K}\Psi_h\phi_h\left(1+\bar{x}_h+\nu_{hi}(x)\right)\right)^2}$$

The estimated MSE of τ_{ASi} = 1,2,3 denoted by $M\hat{S}E\left(\tau_{ASi}\right)$ is given as:

$$M\hat{S}E\left(\tau_{ASi}\right) = \mathrm{v}\left(\bar{y}_{st}\right) + \sigma_{st}^{*^2}\,\mathrm{v}\left(\bar{x}_{st}\right) - 2\sigma_{st}^*\,\mathrm{cov}\left(\bar{y}_{st}\bar{x}_{st}\right) \tag{31}$$

Also, substituting $\phi_h = \left(1+\bar{x}_h+\nu_{ih}(x)\right)^{-1}$, and $v_{hi}(x)$ be either $g_{Mh}(x)$ or $d_{Mh}(x)$ or $p_{Mh}(x)$, we obtained new estimators as:

$$\left.\begin{aligned}\tau_{AS1}&=\sum_{h=1}^{K}\Psi_h\bar{y}_h+\sigma_{st1}^{*}\left(\sum_{h=1}^{K}\Psi_h\left(1+\bar{X}_h+G_{Mh}(x)\right)-\sum_{h=1}^{K}\Psi_h\left(1+\bar{x}_h+g_{Mh}(x)\right)\right)\\ \tau_{AS2}&=\sum_{h=1}^{K}\Psi_h\bar{y}_h+\sigma_{st2}^{*}\left(\sum_{h=1}^{K}\Psi_h\left(1+\bar{X}_h+D_{Mh}(x)\right)-\sum_{h=1}^{K}\Psi_h\left(1+\bar{x}_h+d_{Mh}(x)\right)\right)\\ \tau_{AS3}&=\sum_{h=1}^{K}\Psi_h\bar{y}_h+\sigma_{st3}^{*}\left(\sum_{h=1}^{K}\Psi_h\left(1+\bar{X}_h+P_{Mh}(x)\right)-\sum_{h=1}^{K}\Psi_h\left(1+\bar{x}_h+p_{Mh}(x)\right)\right)\end{aligned}\right\} \quad (32)$$

where

$$\sigma_{sti}^{*}=\frac{\sum_{h=1}^{K}\Psi_h\left(1+\bar{x}_h+v_{hi}(x)\right)^{-1}\sum_{h=1}^{K}\Psi_h\bar{y}_h-\sum_{h=1}^{K}\Psi_h\left(1+\bar{x}_h+v_{hi}(x)\right)^{-1}\bar{y}_h}{\sum_{h=1}^{K}\Psi_h\left(1+\bar{x}_h+v_{hi}(x)\right)^{-1}\sum_{h=1}^{K}\Psi_h\left(1+\bar{x}_h+v_{hi}(x)\right)-1},$$

$i=1,2,3$

2.3. Properties of the new weights Θ_{hi}^{*} and $\mathrm{H}_{hi}^{*}, i=1,2,3$

Theorem 1: The proposed weights Θ_{hi}^{*} and $\mathrm{H}_{hi}^{*}, i=1,2,3$ are consistent.

Proof: As $n_n \to N_h$, $\bar{x}_h \approx \bar{X}_h$ and $v_{hi}(x) \approx V_{hi}(x)$. Then, the expressions λ_1^{opt} and λ_2^{opt} in $\Theta_{hi}^{*}, i=1,2,3$ converged to zeros and expressions μ_1^{opt} and μ_2^{opt} in $\mathrm{H}_{hi}^{*}, i=1,2,3$ tend to zeros. So,

$$\lim_{n_h\to N_h}\frac{\Theta_{hi}^{*}}{\Psi_h}=1 \quad (33)$$

$$\lim_{n_h\to N_h}\frac{\mathrm{H}_{hi}^{*}}{\Psi_h}=1 \quad (34)$$

Theorem 2: $\lim\limits_{n_h\to N_h}\sum_{h=1}^{K}\Theta_{hi}^{*}=1$ and $\lim\limits_{n_h\to N_h}\sum_{h=1}^{K}\mathrm{H}_{hi}^{*}=1$.

Proof: Take the summation of Θ_{hi}^{*} and $\mathrm{H}_{hi}^{*}, i=1,2,3$ over K, we obtained

$$\sum_{h=1}^{K}\Theta_{hi}^{*}=1+\lambda_1^{opt}\sum_{h=1}^{K}\Psi_h\phi_h\left(\bar{x}_h+v_{hi}(x)\right)+\lambda_2^{opt}\sum_{h=1}^{K}\Psi_h\phi_h \quad (35)$$

$$\sum_{h=1}^{K}\mathrm{H}_{hi}^{*}=1+\mu_1^{opt}\sum_{h=1}^{K}\Psi_h\phi_h\left(1+\bar{x}_h+v_{hi}(x)\right)+\mu_2^{opt}\sum_{h=1}^{K}\Psi_h\phi_h, \quad (36)$$

Take the limits $n_n \to N_h$ of Eqs. (35) and (36), $\lambda_1^{opt} \approx 0, \lambda_2^{opt} \approx 0, \mu_1^{opt} \approx 0, \mu_2^{opt} \approx 0$, $\overline{x}_h \approx \overline{X}_h$, $v_{hi} \approx V_{hi}$, hence the proof.

Theorem 3: $0 < \Theta_{hi}^* < 1$ and $0 < \mathrm{H}_{hi}^* < 1, i = 1,2,3$.

Proof: As $n_n \to N_h$, $\lambda_1^{opt} \approx 0, \lambda_2^{opt} \approx 0, \mu_1^{opt} \approx 0, \mu_2^{opt} \approx 0$, then

$$\lim_{n_h \to N_h} \Theta_{hi}^* = \lim_{n_h \to N_h} \mathrm{H}_{hi}^* = \Psi_h = N_h / N \tag{37}$$

Since $N_h > 0$ (population size of stratum h), $N = \sum_{h=1}^{K} N_h > 0$ (Total population under study) and $N_h <, N$, then $0 < \psi_h < 1$, $\left(\psi_h = \dfrac{N_h}{N}\right)$, hence the proof.

3. Simulation Study

We conducted simulation studies to examine the performance of the proposed estimators compared to the usual unbiased estimator, Rao and Khan (2016) estimators and Nidhi et al. (2017) estimators. We generated two sets of data of size 1000 units each as the study populations each stratified into three non-overlapping heterogeneous groups of sizes 200, 300 and 500, respectively. The assumptions about the populations are summarized in Table 1. Samples of sizes 20, 30 and 50 respectively from the three strata are obtained 10,000 times by SRSWOR method from each stratum respectively. The biases and precision (PREs) of the considered estimators are computed using Eqs. (38) and (39) respectively.

$$Bias\left(\hat{\theta}\right) = \frac{1}{10000} \sum_{j=1}^{10000} \left(\hat{\theta} - \overline{Y}\right) \tag{38}$$

$$PRE\left(\hat{\theta}_i\right) = \left(\mathrm{var}\left(\theta\right) / \mathrm{var}\left(\theta_l\right)\right) 100 \tag{39}$$

where $\mathrm{var}\left(\theta\right) = \dfrac{1}{10000} \sum_{j=1}^{10000} \left(\tau_{st} - \overline{Y}\right)^2$,

$$\mathrm{var}\left(\hat{\theta}_l\right) = \frac{1}{10000} \sum_{j=1}^{10000} \left(\hat{\theta}_l - \overline{Y}\right)^2, \; \hat{\theta}_l = \tau_{RK1}, \tau_{RK2}, \tau_{AR1}, \tau_{AR2}, \tau_{NSSS1}, \tau_{NSSS2}, \tau_{AR3}, \tau_{AS1}, \tau_{AS2}, \tau_{AS3}$$

Table1. Population used for Empirical Study

Population	**Auxiliary variable *x***	**Study variable *y***
I	$x_h \sim \exp(\lambda_h)$, λ_1=0.2, λ_2=0.3, λ_3=0.1	$y_{hi} = 50\alpha x_{hi} + \xi_{hi}, h = 1,2,3$ $\alpha = 0.5, 1, 1.5, 2.0, 2.5$ $\xi_h \sim N(\phi_h, \psi_h), \phi_h = 0, \psi_h = 1$,
II		$y_{hi} = \alpha x_{hi} + x_{hi}^2 + \xi_{hi}, h = 1,2,3$ $\alpha = 0.5, 1, 1.5, 2.0, 2.5$ $\xi_h \sim N(\phi_h, \psi_h), \phi_h = 0, \psi_h = 1,$
III		$y_{hi} = \alpha x_{hi} + x_{hi}^2 + x_{hi}^3 + \xi_{hi}, h = 1,2,3$ $\alpha = 0.5, 1, 1.5, 2.0, 2.5,$ $\xi_h \sim N(0,1), h = 1,2,3$
IV	$x_h \sim N(\mu_h, \sigma_h)$, μ_i = 30, μ_2 = 50, μ_3 = 15, σ_1 = 25, σ_2 = 70, σ_3 = 20,	$y_{hi} = 50\alpha x_{hi} + \xi_{hi}, h = 1,2,3$ $\alpha = 0.5, 1, 1.5, 2.0, 2.5$ $\xi_h \sim N(\phi_h, \psi_h), \phi_h = 0, \psi_h = 1,$
V		$y_{hi} = \alpha x_{hi} + x_{hi}^2 + \xi_{hi}, h = 1,2,3$ $\alpha = 0.5, 1, 1.5, 2.0, 2.5$ $\xi_h \sim N(\phi_h, \psi_h), \phi_h = 0, \psi_h = 1,$
VI		$y_{hi} = \alpha x_{hi} + x_{hi}^2 + x_{hi}^3 + \xi_{hi}, h = 1,2,3$ $\alpha = 0.5, 1, 1.5, 2.0, 2.5,$ $\xi_h \sim N(0,1), h = 1,2,3$

Table 2. Biases and PREs of the Proposed and Some Existing Related Estimators using Population I

Estimators	Biases					Percentage Relative Efficiencies (PREs)				
	Values of α					Values of α				
	0.5	**1.0**	**1.5**	**2.0**	**2.5**	**0.5**	**1.0**	**1.5**	**2.0**	**2.5**
τ_{st}	-0.1199	-0.2401	-0.3603	-0.4805	0.2993	100.0	100.0	100.0	100.0	100.0
Rao and Khan (2016)										
τ_{RK1}	-0.4781	-0.9558	-1.4336	-1.9113	-1.3249	149.318	149.325	149.329	149.331	150.369
τ_{RK2}	-0.4283	-0.8564	-1.2845	-1.7125	-1.0867	160.017	160.029	160.033	160.036	161.490
Nidhi et al. (2017)										
τ_{NSSS1}	0.0491	0.0982	0.1473	0.1964	1.1046	156.3678	156.3734	156.3748	156.3754	154.9536
τ_{NSSS2}	0.0089	0.0178	0.0266	0.03542	0.9089	158.5629	158.5731	158.5759	158.5772	157.2624
Proposed										
τ_{AR1}	-0.6866	-1.3760	-2.0654	-2.75485	-2.4211	132.6218	132.5576	132.5372	132.5272	133.3487
τ_{AR2}	-0.26467	-0.5279	-0.7912	-1.05451	-0.2559	166.9257	166.9737	166.9902	166.9984	167.4642
τ_{AR3}	-0.3862	-0.7702	-1.1541	-1.5381	-0.8416	163.2251	163.2689	163.2839	163.2915	163.8406
τ_{AS1}	-0.2495	-0.4973	-0.7452	-0.9931	-0.1819	167.2016	167.2560	167.2745	167.2837	167.6974
τ_{AS2}	-0.2520	-0.5024	-0.7528	-1.0032	-0.1955	167.3924	167.4479	167.4667	167.47616	167.947
τ_{AS3}	-0.3797	-0.7570	-1.1342	-1.5113	-0.8140	63.8849	163.9363	163.9538	163.9625	164.5274

Table 3. Biases and PREs of the Proposed and Some Existing Related Estimators using Population II

Estimators	Biases					Percentage Relative Efficiencies (PREs)				
	Values of α					Values of α				
	0.5	**1.0**	**1.5**	**2.0**	**2.5**	**0.5**	**1.0**	**1.5**	**2.0**	**2.5**
τ_{st}	0.0198	0.1082	0.2235	-0.3659	-0.1963	100.0	100.0	100.0	100.0	100.0
Rao and Khan (2016)										
τ_{RK1}	-1.4229	-1.0521	-1.4653	-0.8038	-0.8264	194.0948	225.4986	208.2264	203.2263	209.8469
τ_{RK2}	-1.3801	-1.0133	-1.3950	-0.8333	-0.8324	174.4426	196.9844	184.5767	180.0022	186.5958
Nidhi et al. (2017)										
τ_{NSSS1}	-2.3714	-1.7997	-2.9904	-1.1611	-1.3231	363.4204	475.658	399.9927	368.2536	379.2177
τ_{NSSS2}	-2.2859	-1.7715	-2.8968	-1.1795	-1.2936	355.0888	463.7691	387.9465	360.372	366.1231
Proposed										
τ_{AR1}	1.3128	-2.5101	-1.4748	-0.7958	-1.6216	415.0217	553.2578	493.2792	464.711	498.6249
τ_{AR2}	-1.1827	1.6689	1.6398	0.2355	0.9310	744.0226	915.3678	939.7388	872.3637	865.1686
τ_{AR3}	1.6517	1.9828	1.3296	0.3689	0.5167	748.1287	911.4257	919.9237	861.708	855.4657
τ_{AS1}	-0.7447	1.5009	1.7761	0.6631	1.4878	762.4458	936.2921	989.2003	916.4124	913.2026
τ_{AS2}	-0.8147	1.3728	1.5680	0.6804	1.5195	748.2332	937.1372	965.174	893.2784	889.9554
τ_{AS3}	1.4591	1.1696	1.8547	0.3237	0.6649	747.7217	931.0807	939.6995	877.5081	877.5759

Table 4. Biases and PREs of the Proposed and Some Existing Related Estimators using Population III

Estimators	Biases					Percentage Relative Efficiencies (PREs)				
	Values of α					Values of α				
	0.5	**1.0**	**1.5**	**2.0**	**2.5**	**0.5**	**1.0**	**1.5**	**2.0**	**2.5**
τ_{st}	-85.250	-33.8447	93.9293	-99.575	45.64163	100.0	100.0	100.0	100.0	100.0
Rao and Khan (2016)										
τ_{RK1}	-70.9348	-72.8609	-104.612	-120.712	-47.6903	145.0381	139.5579	135.677	144.9092	187.7669
τ_{RK2}	-66.3908	-70.8230	-95.5546	-112.462	-44.6518	139.6717	135.0246	131.2441	139.5384	185.1077
Nidhi et al. (2017)										
τ_{NSSS1}	-123.729	-99.4131	-192.752	-223.938	-92.5372	194.2402	180.8727	171.6519	191.2706	187.7669
τ_{NSSS2}	-119.598	-97.9878	-184.089	-218.603	-89.8080	191.1841	177.2509	169.5261	188.8559	185.1077
Proposed										
τ_{AR1}	20.0724	84.6589	82.3238	92.8401	41.9473	204.1579	276.4017	277.1548	237.1817	216.0508
τ_{AR2}	-20.8623	-41.1502	-89.8403	-92.0863	-41.7162	210.4178	303.9027	293.7735	273.5902	250.8872
τ_{AR3}	-59.5318	-75.6810	-89.9305	-99.9197	-32.7882	224.7257	302.0734	291.9818	282.2429	263.1409
τ_{AS1}	-23.322	-41.8021	-84.7031	-95.5647	-41.1903	186.9049	301.6926	288.1884	255.8412	226.3107
τ_{AS2}	-22.3187	-39.3112	-41.1733	-43.4198	-27.078	182.7662	295.0898	281.876	249.3989	221.7451
τ_{AS3}	-50.2655	-111.364	-248.983	-205.499	-124.125	196.329	296.5801	284.352	259.5436	234.5219

Table 5. Biases and PREs of the Proposed and Some Existing Related Estimators using Population IV

Estimators	Biases					Percentage Relative Efficiencies (PREs)				
	Values of α					Values of α				
	0.5	**1.0**	**1.5**	**2.0**	**2.5**	**0.5**	**1.0**	**1.5**	**2.0**	**2.5**
τ_{st}	0.2744885	0.5497394	0.8249903	1.100241	1.375492	100	100	100	100	100
Rao and Khan (2016)										
τ_{RK1}	1.7372	3.4735	5.2098	6.9460	8.6823	177.7244	177.7181	177.7161	177.7150	177.7144
τ_{RK2}	1.9543	3.9079	5.8615	7.8151	9.7687	184.4245	184.4309	184.4329	184.4341	184.4347
Nidhi et al. (2017)										
τ_{NSSS1}	2.5830	3.1666	3.7503	4.3339	4.9176	175.632	175.6379	175.6398	175.6408	175.6413
τ_{NSSS2}	2.5477	3.0961	3.6444	4.1928	4.7412	176.6904	176.6964	176.6983	176.6993	176.6999
Proposed										
τ_{AR1}	0.95250	1.9034	2.8544	3.8053	4.7563	179.8097	179.7991	179.7956	179.7939	179.7929
τ_{AR2}	1.8478	3.6959	5.5439	7.392041	9.2401	186.4118	186.4170	186.4187	186.419	186.4200
τ_{AR3}	3.5021	7.00436	10.5066	14.0089	7.5112	178.4900	178.4933	178.4944	178.4949	178.4952
τ_{AS1}	1.8215	3.6434	5.4653	7.2871	9.1089	186.5754	186.5808	186.5822	186.5831	186.5837
τ_{AS2}	1.8432	3.6867	5.5302	7.3737	9.217267	186.6404	186.6455	186.6471	186.6479	186.6485
τ_{AS3}	1.4689	3.9379	5.4069	6.8759	7.3450	78.7401	178.7433	178.7443	178.7448	178.7452

Table 6. Biases and PREs of the Proposed and Some Existing Related Estimators using Population V

Estimators	Biases					Percentage Relative Efficiencies (PREs)				
	Values of α					Values of α				
	0.5	**1.0**	**1.5**	**2.0**	**2.5**	**0.5**	**1.0**	**1.5**	**2.0**	**2.5**
τ_{st}	6.56038	6.550943	6.541506	6.532068	6.522631	100.0	100.0	100.0	100.0	100.0
Rao and Khan (2016)										
τ_{RK1}	13.32939	13.32939	13.32939	13.32939	13.32939	91.30676	91.86576	92.42797	92.9934	93.56203
τ_{RK2}	25.09506	25.09506	25.09506	25.09506	25.09506	95.2063	95.7892	96.3754	96.9649	97.5579
Nidhi et al. (2017)										
τ_{NSSS1}	2.051284	2.002373	1.953462	1.904551	1.85564	214.811	216.178	217.554	218.938	220.329
τ_{NSSS2}	0.8098365	0.754982	0.700128	0.645274	0.590419	215.332	216.691	218.059	219.434	220.817
Proposed										
τ_{AR1}	-36.61819	-36.8667	-37.1151	-37.3636	-37.6121	487.572	486.169	484.733	483.263	481.762
τ_{AR2}	2.630642	2.816348	3.002054	3.187761	3.373467	665.024	664.085	663.020	661.834	660.529
τ_{AR3}	112.4176	113.2183	114.0189	114.8196	115.6202	442.691	441.170	439.603	437.991	436.336
τ_{AS1}	-0.040959	0.138177	0.317312	0.496448	0.67558	795.44	793.314	791.015	788.549	785.924
τ_{AS2}	3.058078	3.248598	3.439118	3.629638	3.820158	655.1993	654.1062	652.8923	651.5614	650.1173
τ_{AS3}	113.2075	114.0165	114.8255	115.6345	116.4435	435.7339	434.1481	432.5177	430.8449	429.1322

Table 7. Biases and PREs of the Proposed and Some Existing Related Estimators using Population VI

Estimators	Biases					Percentage Relative Efficiencies (PREs)				
	Values of α					Values of α				
	0.5	**1.0**	**1.5**	**2.0**	**2.5**	**0.5**	**1.0**	**1.5**	**2.0**	**2.5**
τ_{st}	895.8263	895.8169	895.8075	895.798	895.7886	100.0	100.0	100.0	100.0	100.0
Rao and Khan (2016)										
τ_{RK1}	-5881.96	-5881.96	-5881.96	-5881.96	-5881.96	172.7992	172.8052	172.8113	172.8174	172.824
τ_{RK2}	-4105.46	-4105.46	-4105.46	-4105.46	-4105.46	182.5521	182.5585	182.5649	182.5713	182.578
Nidhi et al. (2017)										
τ_{NSSS1}	-3258.36	-3258.41	-3258.46	-3258.51	-3258.56	175.5434	175.5492	175.5551	175.5609	175.567
τ_{NSSS2}	-3242.38	-3242.44	-3242.49	-3242.55	-3242.61	172.8302	172.8358	172.8414	172.847	172.8525
Proposed										
τ_{AR1}	-7433.06	-7433.31	-7433.55	-7433.80	-7434.05	253.0657	253.0641	253.0626	253.061	253.0595
τ_{AR2}	-6218.15	-6217.97	-6217.78	-6217.59	-6217.41	305.5811	305.6013	305.6216	305.6418	305.6621
τ_{AR3}	3625.091	3625.891	3626.692	3627.493	3628.293	294.7363	294.7562	294.776	294.7959	294.8157
τ_{AS1}	-6197.92	-6197.74	-6197.57	-6197.39	-6197.21	295.649	295.6671	295.6852	295.7033	295.7213
τ_{AS2}	-6210.93	-6210.74	-6210.55	-6210.36	-6210.17	306.1578	306.1782	306.1986	306.219	306.2395
τ_{AS3}	3648.493	3649.302	3650.111	3650.92	3651.729	295.2923	295.3123	295.3324	295.3524	295.3724

4. Discussion

Tables 2, 3, 4, 5, 6 and 7 showed the results of biases and PREs of the usual unbiased, Rao and Khan (2016) and Nidhi et al. (2017) and proposed calibration estimators using populations I, II, III, IV, V and VI respectively defined in Table 1 for different values of α = (0.5, 1.0, 1.5, 2.0, 2.5). The results of the PREs in Table 2 revealed that for all the values of α (coefficients of linear component of response variable models) using linear function, the proposed estimators have highest values except the proposed estimator τ_{AR1} performed below Rao and Khan (2016) and Nidhi et al. (2017) estimators under normal distribution while the results of Table 5 revealed that for all the values of α (coefficients of linear component of response variable models) in the linear function, the proposed estimators have highest values except the proposed estimators τ_{AR1}, τ_{AR2}, τ_{AR3} which performed below Rao and Khan (2016) τ_{RK2} estimator under exponential distribution. Also, the results of the PREs in Tables 3, 4, 6, and 7 revealed that for all the values of α (coefficients of linear component of study (response) variable models) using linear, quadratic and cubic functions in Table 1 for both normal and exponential distributions, the proposed estimators have highest values except some few cases in which the proposed estimators τ_{AS1} and τ_{AS2} performed below Nidhi et al. (2017). These results implied that the proposed estimators on the average are more efficient in estimation of population mean than other related estimators considered in this study.

5. Conclusion

In this study, we used auxiliary character for a heterogeneous population in the form of robust statistical measures based on Gini's mean difference, Downton's method and probability weighted moments. These measures which are not unduly affected by outliers present in the data and provide more efficient estimates of population parameters in the presence of extreme values were used as alternatives for coefficient of variation used by Rao and Khan (2016). From the results of the Tables 2 and 3, it is observed that the estimators proposed under both the calibration schemes are not only robust against outliers but more efficient than usual ratio estimator in stratified sampling.

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